AN ANALOGUE OF A THEOREM OF KURZWEIL

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Abstract. A theorem of Kurzweil (’55) on inhomogeneous Diophantine approximation states that if \( \theta \) is an irrational number, then the following are equivalent: (A) for every decreasing positive function \( \psi \) such that \( \sum_{q=1}^{\infty} \psi(q) = \infty \), and for almost every \( s \in \mathbb{R} \), there exist infinitely many \( q \in \mathbb{N} \) such that \( \| q\theta - s \| < \psi(q) \), and (B) \( \theta \) is badly approximable. This theorem is not true if one adds to condition (A) the hypothesis that the function \( q \mapsto q\psi(q) \) is decreasing. In this paper we find a condition on the continued fraction expansion of \( \theta \) which is equivalent to the modified version of condition (A). This expands on a recent paper of D. H. Kim (’14).